Research Statement

The core of my research is in nonlinear partial differential equations (PDEs). I apply functional analytic methods to study PDEs in a rigorous mathematical framework. Since my early education in physics, I have always been fascinated by the diversity of wave motions in Nature. My research is now mainly focused on nonlinear PDEs describing wave phenomena in various contexts. I prove theorems about existence and qualitative properties (e.g. regularity, stability, blow-up) of such waves. In recent years, I have studied various important PDEs coming from physics. For instance, models of large-scale oceanic waves based on the *Euler equation* [9], phase transitions in *liquid crystals* [1], or properties of the pressure in solitary *water waves* [10]. But my main research interests are in *nonlinear Schrödinger (NLS) equations*, which arise in the modelling of a variety of physical phenomena, including the propagation of light in nonlinear optical media (e.g. optical fibres), Langmuir waves in plasma, Bose–Einstein condensates, or water waves. A special focus is on NLS equations with a nontrivial spatial dependence, which govern wave propagation in inhomogeneous media and are often referred to as inhomogenous NLS (INLS).

NLS equations belong to the broader class of *nonlinear dispersive equations*. Their asymptotics depend on a competition between linear (dispersive) effects and nonlinear (focusing) effects. The global dynamics can be roughly classified into three qualitatively different asymptotic behaviours: scattering, finite time blow-up, or soliton. *Scattering* occurs when dispersion prevails over nonlinear effects and the waves behave as linear waves for large times. On the contrary, when nonlinear effects beat dispersion, a solution may form a singularity and *blow up* (in a suitable function space norm) in finite time. The *solitons* realise the perfect balance between dispersion and focusing; they are stationary (or travelling) localised structures retaining the same shape at all times.

I currently have several ongoing projects devoted to various aspects of the global dynamics of NLS equations. Following recent progress on finite time blow-up solutions for INLS [3], I am pursuing work in this direction with Elek Csobo, my PhD student at TU Delft. I also have a joint project with Vladimir Georgiev (Pisa) and Jacopo Bellazzini (Sassari) on scattering for INLS with singular coefficients. An upcoming visit to Thomas Duyckaerts (Paris 13) will also be focused on scattering for INLS. Blow-up and scattering solutions of INLS equations will be central objects of focus of my research in the next few years.

A large part of my earlier work on NLS equations was concerned with stability of solitons for equations arising naturally in nonlinear optics, having spatial inhomogeneities and/or nonlinear terms more complicated than the pure-power nonlinearity, see [4–8]. Solitons play a pivotal role in the global dynamics and a deep understanding of their stability properties (gained through variational and bifurcation methods) gave me original insight into the influence of spatial modulations on the dynamics, which guided me in designing the aforementioned projects.

Other advanced/long-term collaborations on dispersive equations include a project with Elek Csobo, Stefan Le Coz and Julien Royer (Toulouse) about stability of solitons for inhomogeneous nonlinear Klein–Gordon equations, and a project about stability of solitons for systems of NLS with Stephan de Bièvre (Lille), Stefan Le Coz and Simona Rota Nodari (Dijon), based on the general approach to stability developed in [2]. These projects are partly funded by French/Dutch grants CIMI and Van Gogh obtained very recently.

Beside NLS dynamics, I plan to continue the programme initiated in [1], where Bachmann and I derived continuum limits for nematic liquid crystals, and described their phase transitions based on effective integral equations governing the macroscopic system. The next step will be to include spatial variations in our model which, for now, only concerns homogeneous systems. We restricted our attention to equilibrium states so far, but a long term goal is to derive hydrodynamic limits for these systems. I will also pursue the study of water waves in several directions, including geophysical waves [9], and the investigation by bifurcation methods of water waves with non-zero vorticity.

References

[1] S. Bachmann, F. Genoud, Scaling limits and phase transitions for nematic liquid crystals in the continuum, under review, preprint http://arxiv.org/abs/1508.05025

[2] S. de Bièvre, F. Genoud and S. Rota Nodari, Orbital stability: analysis meets geometry, in: C. Besse, J. C. Garreau (eds.), *Nonlinear Optical and Atomic Systems*, Lecture Notes in Mathematics 2146, Springer, 2015, pp. 147–273.

[3] V. Combet, F. Genoud, Classification of minimal mass blow-up solutions for an L^2 critical inhomogeneous NLS, *J. Evol. Equ.*, **16** (2016), 483–500.

[4] F. Genoud, Existence and orbital stability of standing waves for some nonlinear Schrödinger equations, perturbation of a model case, *J. Differential Equations* **246** (2009), 1921–1943.

[5] F. Genoud, Bifurcation and stability of travelling waves in self-focusing planar waveguides, *Adv. Nonlinear Stud.* **10** (2010), 357–400.

[6] F. Genoud, A smooth global branch of solutions for a semilinear elliptic equation on $\mathbb{R}^N$, *Calc. Var. Partial Differential Equations* **38** (2010), 207–232.

[7] F. Genoud, Orbitally stable standing waves for the asymptotically linear one-dimensional NLS, *Evolution Equations and Control Theory* **2** (2013), 81–100.

[8] F. Genoud, B.A. Malomed and R.M. Weishäupl, Stable NLS solitons in a cubic-quintic medium with a delta-function potential, *Nonlinear Anal.* **133** (2016), 28–50.

[9] F. Genoud, D. Henry, Instability of equatorial water waves with an underlying current, *J. Math. Fluid Mech.* **16** (2014), 661–667.

[10] F. Genoud, Extrema of the dynamic pressure in a solitary wave, *Nonlinear Anal.* **155** (2017), 65–71.